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xthst: Testing for slope homogeneity in Stata

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Abstract. This article introduces a new community contributed Stata command, `xthst`, to test for slope homogeneity in panels with a large number of observations over cross sectional units and time periods. The program implements such a test, the delta test (Pesaran and Yamagata 2008). Under its null, slope coefficients are homogeneous across cross-sectional units. Under the alternative, slope coefficients are heterogeneous in the cross-sectional dimension. `xthst` also includes two extensions. The first is a heteroskedasticity auto-correlation robust test on the lines of Blomquist and Westerlund (2013). The second extension is a cross-sectional dependence robust version. All tests are discussed and examples using a Solow growth model are presented. A Monte Carlo simulation shows that the size and the power behave as expected.

Keywords: st0001, parameter heterogeneity, fixed effects, pooled OLS, mean-group estimator, cross section dependence

1 Introduction

Today, panel data is widly used for empirical studies in several research areas and the benefits of panel data are well known (Baltagi 2013, p. 6). Linear regression is undoubtedly the workhorse in empirical research and graduate textbooks like Angrist and Pischke (2009, p. 86), encourage researchers to use regression models. Standard panel data regression models like *fixed-effect* (FE) and *random effects* (RE) all assume that the parameter of interest is homogeneous. Incorrectly ignoring slope heterogeneity might bias the results Pesaran and Smith (1995). Whether the homogeneity assumption holds, needs to be clarified before turning to the underlying empirical question.

One possibility for testing slope homogeneity is to apply the F-test on the difference of the sum of squared residuals from pooled ordinary least squares (OLS) and cross-section unit specific OLS regression (Baltagi 2013, p. 64). The main drawback from the latter test is the homoskedastic error variance assumption. In addition, the F-test is based on fixed N assumptions and the test is shown to perform poor unless $T > N$ (Bun 2004). Such $T > N$ panels are relatively rare and in the empirical literature often not used. Pesaran et al. (1996) proposed a Hausman-type test for $N > T$ comparing the FE estimator and cross-section unit specific OLS, but the procedure is not applicable to panel data models with only strictly exogenous regressors or autoregressive models (Pesaran and Yamagata 2008).

In this article, we introduce a new community contributed command, `xthst`, which implements a test for slope homogeneity in the large N and T case, where N can be rela-

 st0001

tive large to T. The command implements the test presented by Pesaran and Yamagata (2008), producing a normal distributed test statistic under the null hypothesis of homogeneous slope coefficients. The concept of the test is to compare the distance between coefficients obtained by a pooled fixed effects regression and by a cross-sectional unit specific regression. The difference is weighted by the unit specific standard errors and thus allows for residual heteroskedasticity. Blomquist and Westerlund (2016) proposed a heteroskedasticity and autocorrelation (HAC) consistent extension, which is included in the `xthst` package. In addition, `xthst` offers a cross-sectional dependence robust test statistic which partials out cross-section averages, inspired by Pesaran (2006) and Chudik and Pesaran (2015b). The latter technique has not been derived in a theory, however Monte Carlo simulations show that this approach seems to be promising.

To our knowledge there are two recent studies developing tests for slope homogeneity when the errors are cross-sectional dependent. Using the same framework as Pesaran and Yamagata (2008), Ando and Bai (2015) presented a test that utilises the initial idea of the *interactive-effect estimator* (Bai 2009). In the latter setup, the number of unknown common factors causing cross-sectional dependence need to be known or estimated. Choosing a different approach, Blomquist and Westerlund (2016) developed a bootstrap based test.

For the remainder of this article, we use the following notation: $x_{i,t}$ refers to a scalar. Lower case bold letters, such as $\mathbf{x}_{i,t}$, denote a vector, usually in a $1 \times k$ dimension, where k refers to the number of regressors. Matrices are denoted in bold and uppercase, such as $\mathbf{X}_i$. Cross-sectional units are denoted by i or j and time periods by t. The number of cross-sectional units is N, whereas the number of time periods is T. Finally, squared brackets describe the *floor* of a number, for example $[T^{1/3}] = [100^{1/3}] = [4.64] = 4$.

This article is arranged as follows. In section 2 we review and discuss the econometric theory for the different tests. In Section 3, the `xthst` syntax and the available options are described. Examples using Solow-style growth models are presented in Section 4. Section 5 contains a detailed description of the Monte Carlo simulation setup alongside results used for assessing the finite sample properties. We close the article with a conclusion.

2 Econometric Theory

Consider the classical panel data model with heterogeneous slopes

$$y_{i,t} = \mu_i + \boldsymbol{\beta}'_{1i}\mathbf{x}_{1i,t} + \boldsymbol{\beta}'_{2i}\mathbf{x}_{2i,t} + \varepsilon_{i,t}, \qquad (1)$$

where $i = 1, .., N$ represents the cross-sectional dimension and $t = 1, .., T$ the time dimension. $\boldsymbol{\beta}_{1i}$ is $k_1 \times 1$ and $\boldsymbol{\beta}_{2i}$ is $k_2 \times 1$ are vectors of unknown slope coefficients with $k = k_1 + k_2$ being the total number of regressors. $\mathbf{x}_{1i,t}$ is a $k_1 \times 1$ vector and $\mathbf{x}_{2i,t}$ a $k_2 \times 1$ vector containing strictly exogenous regressors. The null hypothesis of interest is formulated as:

$$H_0 : \boldsymbol{\beta}_{2i} = \boldsymbol{\beta}_2 \text{ for some } i, \tag{2}$$

against the alternative:

$$H_A : \boldsymbol{\beta}_{2i} \neq \boldsymbol{\beta}_2 \text{ for some } i \neq j. \tag{3}$$

Only coefficients in $\boldsymbol{\beta}_{2i}$ are of interest and tested for slope homogeneity. The remaining coefficients $\boldsymbol{\beta}_{1i}$ are assumed to be heterogeneous, $\boldsymbol{\beta}_{1i} \neq \boldsymbol{\beta}_1$. In the extreme case if all coefficients are under inspection, $\mathbf{x}_{1i,t}$ reduces to zero variables and $k = k_2$. For a matter of clarity, we use the sub-set notation for the remainder of the article.

2.1 The Standard delta test

Based on a standardised version of Swamy's test (Swamy 1970), Pesaran and Yamagata (2008) proposed a test for slope homogeneity for panel data with large N and T. The test assumes that $\varepsilon_{i,t}$ and $\varepsilon_{j,s}$ are independently distributed for $i \neq j$ and/or $t \neq s$, but allows for a heterogeneous variance. The test statistic is given by

$$\tilde{\Delta} = \frac{1}{\sqrt{N}} \left(\frac{\sum_{i=1}^{N} \tilde{d}_i - k_2}{\sqrt{2k_2}} \right), \tag{4}$$

where the statistic, under H_0 in Eq. (2), is asymptotically $\tilde{\Delta} \sim \mathcal{N}(0,1)$. In Eq. (4), $\tilde{d}_i$ is defined as the weighted difference between the cross-sectional unit specific estimate and the pooled estimate:

$$\tilde{d}_i = (\hat{\boldsymbol{\beta}}_{2i} - \tilde{\boldsymbol{\beta}}_{2WFE})' \frac{\mathbf{X}_{2i}' \mathbf{M}_{1i} \mathbf{X}_{2i}}{\tilde{\sigma}_i^2} (\hat{\boldsymbol{\beta}}_{2i} - \tilde{\boldsymbol{\beta}}_{2WFE}), \tag{5}$$

where $\mathbf{X}_{2i} = (\mathbf{x}_{2i,1}, ..., \mathbf{X}_{2i,T_i})'$, $\mathbf{M}_{1i} = \mathbf{I}_{T_i} - \mathbf{Z}_{1i}(\mathbf{Z}_{1i}'\mathbf{Z}_{1i})^{-1}\mathbf{Z}_{1i}'$ and $\mathbf{Z}_{1i} = (\boldsymbol{\tau}_{T_i}, \mathbf{X}_{1i})$ with $\boldsymbol{\tau}_{T_i}$ being a $T_i \times 1$ vector of unity, representing the constant. The coefficients $\hat{\boldsymbol{\beta}}_{2i}$ and $\hat{\boldsymbol{\beta}}_{2WFE}$ are defined as:

$$\hat{\boldsymbol{\beta}}_{2i} = (\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{X}_{2i})^{-1}\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{y}_i,$$
$$\hat{\boldsymbol{\beta}}_{2WFE} = \left(\sum_{i=1}^{N} \frac{\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{X}_{2i}}{\tilde{\sigma}_i^2} \right)^{-1} \sum_{i=1}^{N} \frac{\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{y}_i}{\tilde{\sigma}_i^2},$$

where $\mathbf{y}_i = (y_{i,1}, ..., y_{i,T_i})$,

$$\tilde{\sigma}_i^2 = \frac{(\mathbf{y}_i - \mathbf{X}_{2i}\hat{\boldsymbol{\beta}}_{FE})'\mathbf{M}_{1i}(\mathbf{y}_i - \mathbf{X}_{2i}\hat{\boldsymbol{\beta}}_{FE})}{T_i - 1},$$

and

$$\hat{\boldsymbol{\beta}}_{FE} = \left(\sum_{i=1}^{N} \mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{X}_{2i}\right)^{-1} \sum_{i=1}^{N} \mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{y}_i.$$

The regressors which are not of interest, including the constant α_i, are assumed to be heterogeneous, are collected in $\mathbf{Z}_{1i}$ and partialled out using the projection matrix $\mathbf{M}_{1i}$. The asymptotic properties of $\tilde{\Delta}$ are based on $(N,T) \xrightarrow{j} \infty$, such that $\sqrt{N}/T^2 \to 0$. The results presented by Pesaran and Yamagata (2008) also holds if (1) is changed to a standard first order autoregressive model. However, for the latter the N and T are required to jointly go to infinity with the same speed, thus $(N,T) \xrightarrow{j} \infty$ and $N/T \to \kappa$.

In the case of normally distributed errors the mean-variance bias adjusted $\tilde{\Delta}$ can be expressed the following way:

$$\tilde{\Delta}_{adj} = \sqrt{N}\left(\frac{N^{-1}\sum_{i=1}^{N}\tilde{d}_i - k_2}{\sqrt{Var(\tilde{z}_{i,T_i})}}\right), \tag{6}$$

where

$$Var(\tilde{z}_{i,T_i}) = \frac{2k_2(T_i - k - 1)}{T_i - k_1 + 1}.$$

2.2 A HAC robust test

Based on Pesaran and Yamagata (2008), Blomquist and Westerlund (2013) presented an HAC consistent extension. The HAC robust test statistic is given by

$$\tilde{\Delta}_{HAC} = \sqrt{N}\left(\frac{N^{-1}S_{HAC} - k_2}{\sqrt{2k_2}}\right), \tag{7}$$

where

$$S_{HAC} = \sum_{i=1}^{N} T_i(\hat{\boldsymbol{\beta}}_{2i} - \hat{\boldsymbol{\beta}}_{2HAC})'(\hat{\mathbf{Q}}_{i,T_i}\hat{\mathbf{V}}_{i,T_i}^{-1}\hat{\mathbf{Q}}_{i,T_i})(\hat{\boldsymbol{\beta}}_{2i} - \hat{\boldsymbol{\beta}}_{2HAC})$$

$$\hat{\boldsymbol{\beta}}_{2HAC} = \left(\sum_{i=1}^{N} T_i\hat{\mathbf{Q}}_{i,T_i}\hat{\mathbf{V}}_{i,T_i}^{-1}\hat{\mathbf{Q}}_{i,T_i}\right)^{-1} \sum_{i=1}^{N} \hat{\mathbf{Q}}_{i,T_i}\hat{\mathbf{V}}_{i,T_i}^{-1}\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{y}_i,$$

where $\hat{\boldsymbol{\beta}}_{2i}$ again is the OLS estimator for each i, $\mathbf{M}_{1i}$ as described above and $\hat{\mathbf{Q}}_{i,T_i} = T_i^{-1}(\mathbf{X}'_{2i}\mathbf{M}_{1i}\mathbf{X}_{2i})$. The HAC correction is done with the following estimator

$$\hat{\mathbf{V}}_{i,T_i} = \hat{\boldsymbol{\Omega}}_i(0) + \sum_{j=1}^{T_i-1} \kappa(j/B_{i,T_i})[\hat{\boldsymbol{\Omega}}_i(j) + \hat{\boldsymbol{\Omega}}_i(j)'], \tag{8}$$

where $\hat{\boldsymbol{\Omega}}_i(j) = T_i^{-1}\sum_{t=j+1}^{T_i} \hat{\check{\mathbf{u}}}_{i,t}\hat{\check{\mathbf{u}}}'_{i,t-j}$, and $\hat{\check{\mathbf{u}}}_{i,t} = (\check{\mathbf{x}}_{2i,t} - \bar{\check{\mathbf{x}}}_{2i,t})\hat{\varepsilon}_{i,t}$ with $\bar{\check{\mathbf{x}}}_{2i,t} = T_i^{-1}\sum_{t-1}^{T_i} \check{\mathbf{x}}_{2i,t}$ where $\check{\mathbf{x}}_{2i,t}$ is t-th element of $\mathbf{X}_{2i}\mathbf{M}_{1i}$. $\hat{\varepsilon}_{i,t}$ is an estimated residual from a standard fixed-effect regression using $\mathbf{M}_{1i}$ as the projection matrix. In Eq. (8), κ is a kernel function and B_{i,T_i} its bandwidth parameter. Kernels and bandwidths available in `xthst` are discussed in Section 3.2.

2.3 A cross-sectional dependence robust test

Especially in panels with a large number of cross-sectional units and time periods dependence across cross-sectional units can arise. The literature differentiates between weak and strong cross-sectional dependence (Chudik et al. 2011). Weak cross-sectional dependence is often approximated by spatial methods. Strong cross-sectional dependence is modelled by a common time specific factor f_t and factor loading γ_i. The common factors affect all cross-sectional units:

$$\begin{aligned} y_{i,t} &= \mu_i + \boldsymbol{\beta}'_{1i}\mathbf{x}_{1i,t} + \boldsymbol{\beta}'_{2i}\mathbf{x}_{2i,t} + u_{i,t}, \\ u_{i,t} &= \boldsymbol{\gamma}'_i\mathbf{f}_t + \varepsilon_{i,t}, \end{aligned} \tag{9}$$

where $\mathbf{f}_t$ is an $m \times 1$ vector of unknown common factors and $\boldsymbol{\gamma}_i$ is an $mx \times 1$ vector of unknown factor loadings.

In the case of correlation between the common factor and the explanatory variables, leaving the common factors out can lead to an omitted variable bias. The common factors can either be approximated by principal components (Bai 2009) or cross-sectional averages (Pesaran 2006). The approach by Pesaran (2006), the so called common correlated effects (CCE) estimator, has the advantage that the number of common factors does not need to be known in advance. Therefore in the remainder opt for the latter technique for removing strong cross-sectional dependence.

Chudik and Pesaran (2015b) derive a version for weakly exogenous regressors by adding p_{CSA} lags of the cross-sectional averages and recommend setting $p_{CSA} = [T^{1/3}]$. Equation (9) with cross-sectional averages would then be:

$$y_{i,t} = \mu_i + \boldsymbol{\beta}_{1i}'\mathbf{x}_{1i,t} + \boldsymbol{\beta}_{2i}'\mathbf{x}_{2i,t} + \sum_{l=1}^{p_{CSA}} \gamma_{i,l}\bar{\mathbf{v}}_t + \varepsilon_{i,t} \tag{10}$$

$$\bar{\mathbf{v}}_t = \frac{1}{N}\sum_{j=1}^{N}(\mathbf{x}_{1j,t}, \mathbf{x}_{2j,t}, \mathbf{Y}_{j,t}) \tag{11}$$

where $\bar{\mathbf{v}}_t$ are the cross-sectional averages and either $\mathbf{x}_{1i,t}$ or $\mathbf{x}_{2i,t}$ include the lag of the dependent variable. The CCE estimator can be applied to a pooled (CCE-P) and a mean group model (CCE-MG). Therefore the existing delta test can easily be extended to encompass cross-sectional averages and give guidance on whether to use a pooled or mean group model.

For the cross-sectional dependence robust delta test we propose to partial the cross-sectional averages out to remove strong cross-sectional dependence from the model. Assume that matrix $\bar{\mathbf{V}}_t$ contains the cross-sectional averages and their lags. Then the partialling out is done by:

$$\begin{aligned}
\tilde{\mathbf{V}}_t &= \frac{1}{N}\sum_{j=1}^{N}\left(\mathbf{x}_{1j,t}, \mathbf{x}_{2j,t}, y_{j,t}\right), \bar{\mathbf{V}}_t = \left(\tilde{\mathbf{V}}_t, ..., \tilde{\mathbf{V}}_{t-p_{CSA}}\right) \\
\mathbf{M}_{\bar{V}t} &= I_T - \bar{\mathbf{V}}_t\left(\bar{\mathbf{V}}_t'\bar{\mathbf{V}}_t\right)^{-1}\bar{\mathbf{V}}_t' \\
\check{\mathbf{y}}_i &= \mathbf{y}_i\mathbf{M}_{\bar{V}t} \\
\check{\mathbf{X}}_{1i} &= \mathbf{X}_{1i}\mathbf{M}_{\bar{V}t} \text{ and } \check{\mathbf{X}}_{2i} = \mathbf{X}_{2i}\mathbf{M}_{\bar{V}t}
\end{aligned}$$

The defactored variables are then used to construct $\tilde{\Delta}_{CSA}$ following Eq. (4), respectively Eq. (7) for the HAC robust test. Our Monte Carlo simulations show that the test performs sufficiently well, however the test has not been derived in a more theoretical fashion.

3 The `xthst` command

3.1 Syntax

`xthst` *depvar indepvars* [*if*] [`partial(`*varlist_p*`)` `noconstant`

`crosssectional(`*varlist_cr* [`,cr_lags(`*numlist*`)`]`)` `ar` `hac` `bw(`*integer*`)`

`whitening` `kernel(`*kernel_options*`)` `nooutput`]

Data must be [XT] **xtset** before using `xthst`. *depvar* and *indepvars* may contain time-series operators, see [TS] **tsvarlist**.

depvar is the dependent variable of the model to be tested, *indepvars* the independent variables. *varlist_p* are the variables to be partialled out, *varlist_cr* are variables added as cross-sectional averages. `xthst` calculates the cross-sectional averages. *kernel_options* can be *qs*, *bartlett* or *truncated*

Options

`noconstant` suppresses the individual heterogeneous constant, α_i.

`partial(`*varlist_p*`)` requests exogenous regressors in *varlist_p* to be partialled out. The constant is automatically partialled out, if included in the model. Regressors in *varlist* will be included in $\mathbf{z}_{it}$ and are assumed to have heterogeneous slopes, see Section (2).

`ar` allows for an AR(p) model. The degree of freedom of $\tilde{\sigma}^2$ is adjusted. May not be combined with `hac`.

`hac` implements the HAC consistent test by Blomquist and Westerlund (2013). If `kernel` and `bw` are not specified, kernel is set to *bartlett* the data driven bandwidth selection is used, see Section 3.2. May not be combined with `ar`.

`kernel(`*kernel*`)` specifies the kernel function used in calculating the HAC consistent test statistic. Available kernels are *bartlett*, *qs* (quadratic spectral) and *truncated*. Is only required in combination with `hac`.

`bw(#)` sets the bandwith equal to # for the HAC consistent test statistic, where # is an integer greater than zero. Is only required in combination with `hac`. Default is the data driven bandwidth selection, see Section 3.2.

`whitening` performs pre-whitening to reduce small-sample bias in HAC estimation. Is only required in combination with `hac`.

`crosssectional(`*varlist_cr* `[,cr_lags(`*numlist*`)])` defines the variables which are added as cross-sectional averages to the model to approximate strong cross-sectional dependence. Variables in *varlist_cr* are partialled out. `cr_lags(`*numlist*`)` sets the number of lags of the cross-sectional averages. If not defined, but `crosssectional()` contains a *varlist*, then only contemporaneous cross sectional averages are added but no lags. `cr_lags(0)` is the equivalent. The number of lags can variable specific, where the order is the same as defined in `cr()`. For example if `cr(y x)` and only contemporaneous cross-sectional averages of `y` but 2 lags of `x` are added, then `cr_lags(0 2)`.

`nooutput` omits output.

Stored Values

`xthst` stores the following in `r()`:

Scalars

`r(bw)`	bandwith

Macros

`r(cross-sectional)`	variables of which cross-section averages are added
`r(partial)`	variables partialled out
`r(kernel)`	used kernel

Matrices

`r(delta)`	delta and adjusted delta
`r(delta_p)`	p-values of the above

3.2 Kernel and banwidth for the HAC robust test

Three different kernels for the estimation of the variance/covariance matrix when using the HAC robust test are build into `xthst`. The kernels are the *Bartlett*, the *Quadratic spectral* (QS) and the *Truncated* kernel. If the bandwidth is not manually chosen, `xthst` opts for a data-dependent selection based on the chosen kernel. The latter follows Newey and West (1994),

$$B_{i,T_i} = [c(\alpha_i(q)^2 T_i)^{1/(2q+1)}], \tag{12}$$

where scalars c and q depend on the type of kernel. When the Truncated kernel is applied, $\kappa = 1$ and $B_{i,T_i} = [4(T_i/100)^{1/5}]$ (Newey and West 1994). For the QS kernel the parameters are $c = 1.3221$ and $q = 2$, while for the Bartlett kernel $c = 1.1447$ and $q = 1$, see Andrews (1991) and Andrews and Monahan (1992).

In the QS case, $\alpha_i(2)$ follows Andrews (1991),

$$\alpha_i(2) = \sum_{a=1}^{k_2} \frac{4\hat{\rho}_{i,a}^2 \hat{\sigma}_{i,a}^4}{(1-\hat{\rho}_{i,a})^8} \Big/ \sum_{a=1}^{k_2} \frac{\hat{\sigma}_{i,a}^4}{(1-\hat{\rho}_{i,a})^4}. \tag{13}$$

Applying an AR(1) model on $\hat{\mathbf{u}}_{i,t}$ for each i, remembering that $\hat{\mathbf{u}}_{i,t}$ is $k_2 \times 1$, one obtains the estimated autoregressive coefficient $\hat{\rho}_{i,a}$ and variance $\hat{\sigma}_{i,a}^2$ which are used in (13).

For the *Bartlett* kernel case, $\alpha_i(1)$ is estimated according to Newey and West (1994)

$$\alpha_i(1) = \frac{2\sum_{s=1}^{r} s\hat{\sigma}_{i,s}}{\hat{\sigma}_{i,0} + 2\sum_{s=1}^{r}\hat{\sigma}_{i,s}}, \tag{14}$$

where $r = [4(T_i/100)^{2/9}]$ and $\hat{\sigma}_{i,s} = (T_i - 1)^{-1}\sum_{t=j+1}^{T_i} \hat{\mathbf{u}}_{i,t}\hat{\mathbf{u}}_{i,t-j}$.

The `xthst` program offers *prewhitening* to reduce the small-sample bias in a HAC estimation, in line with Blomquist and Westerlund (2013). Applying the *prewhitening* option replaces $\hat{\mathbf{u}}_{i,t}$ above with $\hat{\mathbf{u}}_{i,t}^* = \hat{\mathbf{u}}_{i,t} - \hat{\boldsymbol{\rho}}_i'\hat{\mathbf{u}}_{i,t-1}$, where $\hat{\boldsymbol{\rho}}_i$ is the coefficients from the AR(1) model on $\hat{\mathbf{u}}_{i,t}$ for each i.

4 Examples

In this section we carry out several examples, all draw on a Solow-style growth models using the Penn World Tables (Feenstra et al. 2015). First, we give an example for the standard $\tilde{\Delta}$ test, then we give examples for testing a subset of coefficients, the HAC- and cross-sectional dependence robust extensions.

4.1 Standard $\tilde{\Delta}$ Test

In this section we want to test if the coefficients of a cross-country growth regression are homo- or heterogeneous. To do so, we fit a Solow growth model on the lines of Mankiw et al. (1992), Islam (1995) and Lee et al. (1997). The dependent variable is real GDP per capita in logarithms, `log_rgdpo`. The explanatory variables are human capital, `log_hc`, physical capital, `log_ck`, and population growth added with break even investments of 5%, `log_ngd`. All variables are in logarithms and the dataset contains yearly observations from 1960 until 2007.

For a first exemplified model, we assume a static model, hence no lag of the dependent variable occurs. We want to test if any of the slope coefficients are homo- or heterogeneous. The command line and output are:

```
. xthst d.log_rgdp log_hc log_ck log_ngd
Test for slope homogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
-------------------------------------
         Delta          p-value
          6.328           0.000
 adj.     6.694           0.000
-------------------------------------
Variables partialled out: constant
```

`xthst` automatically assumes a heterogeneous constant. The Delta test statistic is sufficiently large to reject the null of slope homogeneity. Therefore when running this model, an estimator allowing for heterogenous slopes, such as the mean group estimator should be used.

In the next step we add the first lag of GDP, so the regression model is an actual growth model. Extending the command line from above with `L.d.log_rgdpo`:

```
. xthst d.log_rgdp L.d.log_rgdp log_hc log_ck log_ngd
Test for slope homogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
-------------------------------------
         Delta          p-value
          2.957           0.003
 adj.     3.171           0.002
-------------------------------------
Variables partialled out: constant
```

Once again, we can comfortably reject the null at a level of 5%. However we note that the value of the test statistic deceased.

4.2 Testing a subsample

In case the assumption is that all variables except the lag of GDP are heterogeneous, the `partial(varlist_partial)` option can be used. In this case all variables defined `varlist_partial` are partialled out and assumed to be heterogeneous:

```
. xthst d.log_rgdp L.d.log_rgdp log_hc log_ck log_ngd, ///
> partial(log_hc log_ck log_ngd)
Test for slope homogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
─────────────────────────────────────
         Delta          p-value
          2.324           0.020
 adj.     2.409           0.016
─────────────────────────────────────
Variables partialled out: log_hc log_ck log_ngd constant
```

The test confirms that the coefficient of the lag of GDP is heterogeneous. The test statistic decreased in comparison to the model above further.

4.3 Allowing for heteroskedastic and serially correlated errors

In a dynamic macro dataset it is likely that errors exhibit serial correlation. To account for autocorrelation in the residual, the option `hac` can be employed to use the HAC robust standard errors following Blomquist and Westerlund (2013):

```
. xthst d.log_rgdp L.d.log_rgdp log_hc log_ck log_ngd, hac
Test for slope homogeneity
(Blomquist, Westerlund. 2013. Economic Letters)
H0: slope coefficients are homogenous
─────────────────────────────────────
         Delta          p-value
         12.203           0.000
 adj.    13.086           0.000
─────────────────────────────────────
HAC Kernel: bartlett
with average bandwith 3
Variables partialled out: constant
```

The test for slope homogeneity becomes heteroskedastic robust by using a heteroskedastic autocorrelation (HAC) robust estimator for the variance, which relies on a kernel function with a given bandwidth *bw* . The default is to use a *Bartlett* kernel with automatically selected bandwidth following Andrews and Monahan (1992); Newey and West (1994). Besides the *Bartlett* kernel, `xthst` supports the the *Quadratic Spectral* (*QS*) and *Truncated* kernel. The kernels can be set with the option `kernel()` and the bandwidth with `bw`. To use the *QS* kernel with bandwidth 5:

```
. xthst d.log_rgdp L.d.log_rgdp log_hc log_ck log_ngd, ///
> hac bw(5) kernel("qs")
Test for slope homogeneity
(Blomquist, Westerlund. 2013. Economic Letters)
H0: slope coefficients are homogenous
-----------------------------------------------
        Delta        p-value
        -1.843         0.065
 adj.   -1.977         0.048
-----------------------------------------------
HAC Kernel: quadratic spectral (QS)
with average bandwith 5
Variables partialled out: constant
```

4.4 Accounting for Cross-Sectional Dependence

In large panels cross-sectional dependence is likely and mostly unobserved. A popular method to approximate strong cross-sectional dependence is to add cross-sectional averages are further covariates. This estimator is known as the Common Correlated Effects estimator (Pesaran 2006; Chudik and Pesaran 2015b).[1] In Stata the community contributed command `xtdcce2` (Ditzen 2018) introduced the CCE estimator.

On those lines, `xthst` can take out strong cross-sectional dependence by approximating it with cross-sectional averages, thus comparing a common correlated effects pooled and mean group estimator. The cross-sectional averages are partialled out and can be defined by the option `crosssectional(varlist_csa [ , lags(numlist)])`. *varlist_csa* is a variable list containing the variables of which the cross-sectional averages are derived from. The optional *numlist* defines the number of lags of the cross-sectional averages. If not defined, only the base of the cross-sectional averages, the contemporaneous cross-sectional averages are added.

We can use `xtcd2` or `xtcse2` (Ditzen 2018, 2019) to test for weak cross-sectional dependence and estimate the strength of it. The result implies strong cross-sectional dependence for all variables, urgining the inclusion of cross-sectional averages. For the matter of exemplification, we add 1 lag for log GDP, 2 lags for physical capital and 3 for human capital and the break even investments.

```
. xthst d.log_rgdp L.d.log_rgdp log_hc log_ck log_ngd, ///
>  cr(d.log_rgdp log_hc log_ck log_ngd, cr_lags(2 3))
Test for slope homogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
-----------------------------------------------
        Delta        p-value
         3.537         0.000
 adj.    3.903         0.000
-----------------------------------------------
Variables partialled out: constant
Cross Sectional Averaged Variables:  D.log_rgdpo(2) log_hc(3) log_ck(3) log_ngd(3)
```

1. For a summary on cross-sectional dependence see Chudik and Pesaran (2015a).

5 Monte Carlo

In this section we asses the finite sample properties using a Monte Carlo simulation. We focus on the size and the power of the delta test. The simulation set-up follows on the lines of Pesaran and Yamagata (2008); Blomquist and Westerlund (2013), but we add further cross-sectional dependence.

The DGP for the simulation with k regressors is:

$$y_{i,t} = \alpha_i + \sum_{l=1}^{k} \beta_{l,i} x_{i,l,t} + u_{i,t} \tag{15}$$

$$x_{i,l,t} = \alpha_i \left(1 - \rho_{x,i,l}\right) + \rho_{x,i,l} x_{i,l,t-1} + \left(1 - \rho_{x,i,l}\right)^{\frac{1}{2}} v_{i,l,t} \tag{16}$$

The error component $u_{i,t}$ contains serial correlation, if $\rho_{u,i} > 0$, and is heteroskedastic in all specifications. The error components of the independent variables, $v_{i,t}$, are white noise with a unit specific variance and are generated as:

$$u_{i,t} = \rho_{u,i} u_{i,t-1} + \sqrt{1 - \rho_{u,i}^2} \left(\gamma_{u,i} f_t + e_{i,t}\right) \tag{17}$$

$$e_{i,t} \sim N(0, \sigma_{i,e}^2) \text{ with } \sigma_{i,e}^2 = \frac{k\chi^2(2)}{2} \tag{18}$$

$$v_{i,l,t} = \gamma_{x,i,l} f_t + \epsilon_{i,l,t} \tag{19}$$

$$\epsilon_{i,l,t} \sim IIDN(0, \sigma_{\epsilon,i,l}^2) \text{ with } \sigma_{\epsilon,i,l} \sim IID\chi^2(1) \tag{20}$$

The autocorrelation coefficients of the independent variables are generated as $\rho_{x,i,l} \sim IIDU[0.05, 0.95]$. The generation of cross-sectional dependence follows Chudik and Pesaran (2015b) and is introduced by the terms $\gamma_{x,i,l} f_t$ and $\gamma_{u,i} f_t$. The common factors f_t are generated as $f_t = \rho_f f_{t-1} + \xi_t$, $\xi_t \sim IIDN(0, 1 - \rho_f^2)$. ρ_f is varied between 0 (no CSD) and 0.8 (CSD). The factor loadings $\gamma_{x,i,l}$ and $\gamma_{u,i}$ are centred around a common mean:

$$\begin{aligned}
\gamma_{u,i} &= \gamma_u + \eta_{u,i} & \gamma_{x,i,l} &= \gamma_{x,l} + \eta_{x,i,l} \\
\eta_{u,i} &\sim IIDN(0, \sigma_{\gamma,l}^2) & \eta_{x,i,l} &= IIDN(0, \sigma_{x,\gamma,l}^2) \\
\gamma_u &= \sqrt{\frac{1}{m} - \sigma_{y,\gamma,l}^2} & \gamma_{x,l} &= \sqrt{l\left(\frac{2}{m(m+1)} - \frac{2}{m+1}\sigma_{x,\gamma,l}^2\right)} \\
\sigma_{\gamma,l}^2 &= 0.2^2 & \sigma_{x,\gamma,l}^2 &= \sigma_{y,\gamma,l}^2 = 0.2^2,
\end{aligned}$$

with $m = k$ is the number of regressors.

In the case of serial correlated errors, the autocorrelation coefficients of $u_{i,t}$ are generated as $\rho_{u,i} \sim IIDU[0, \rho_u]$, whereas ρ_u is varied between 0 (no serial correlation) and 0.7 (serial correlation).

Specification	ρ_f	ρ_u	Table	Specification	ρ_f	ρ_u	h	Table
k = 1				k = 4				
1	0	0	Table 2	5	0	0	0	Table 7
2	0.8	0	Table 3 & 4	6	0.8	0	0	Table 8
3	0	0.7	Table 5	7	0	0.7	0	Table 9
4	0.8	0.7	Table 6	8	0.8	0.7	0	Table 10
				9	0	0	1	Table 11

Table 1: $\rho_f > 0$ indicates cross-sectional dependence, $\rho_u > 0$ indicates serial correlation in the errors and h is the number of heterogeneous coefficients under the null. For $k = 1, h = 0$.

The main focus of the Monte Carlo simulation exercise will lie on the coefficient $\beta_{l,i}$. Under the null hypothesis of homogeneous slopes, the coefficients are set to unity, $\beta_{l,i} = 1$. Under the alternative, the first $N/2$ coefficients are set to unity, the remaining coefficients are drawn from a normal distribution:

$$\beta_{l,i} = 1 \qquad \text{for } i = 1, ..., \frac{N}{2} \text{ and } l = 1, ..., k \tag{21}$$

$$\beta_{l,i} \sim N(1, 0.04) \qquad \text{for } i = \frac{N}{2} + 1, ..., N \text{ and } l = 1, ..., k \tag{22}$$

For a matter of simplicity, it is assumed that all $k-$coefficients are the same, hence $\beta_{l,i} = \beta_{1i}$. Under the alternative the coefficients are generated as $\beta_{l,i} \sim N(1, 0.04)$ for $i > \frac{N}{2}, l = 1, .., k$. We vary the number of coefficients between $k = 1$ and $k = 4$. In the special case of $k = 4$, the first h coefficients are generated as heterogeneous even under the null hypothesis. These coefficients are then partialled out. We vary h between 0 and 1. The unit specific fixed effect is generated as $\alpha_i \sim N(1, 1)$.

In the simulations we are observe 4 cases, one without serial correlation and cross-sectional dependence, one with either serial-correlation or cross-sectional dependence and a combination of both. We vary the number of regressors between 0 and 4. The different specifications are summarised in Table 1.

5.1 Tests

We are comparing the results for the standard delta test ($\tilde{\Delta}$) and the for heteroskedastic and autocorrelation ($\tilde{\Delta}_{HAC}$) and cross-sectional dependence robust versions ($\tilde{\Delta}_{CSA}$). For cross-sectional dependence robust version $p_{CSA} = [T^{(1/3}]$ lags of the cross-sectional averages are added. This leads to a very small sample in the specification with $T = 20$ and $k = 4$.[2] To avoid problems when calculating the test, we restrict the cross-sectional averages to contain only contemporaneous values $\rho_{CSA} = 0$.

2. In this case there are $p_{CSA} = [20^{1/3}] = 2$ thus $k * (1 + p_{CSA}) + k_{constant} = 13$.

Following Blomquist and Westerlund (2013) the HAC robust delta test performs best with prewhitening and the QS kernel. To confirm their result we compare the bartlett kernel, the non-prewhitened QS, the prewhitend QS and the truncated kernel for specification 2. In order to save space, we focus on the prewhitened delta test with the QS kernel when comparing $\tilde{\Delta}_{HAC(QS)+Whitening}$ to the other two tests. We also employ a mix of the HAC and CSA test for specification 4 and 8.

For the specification without cross-sectional dependence and serially correlated errors, we expect the standard delta test to perform best. For specification 2 the HAC robust and for specification 3 the cross-sectional dependent robust test should show the best size and power. Pesaran and Yamagata (2008) find that an increase in the number of regressors leads to lower performance of the tests. In the special case of $h = 1$ and $k = 4$ we add an additional test, the so-called $\tilde{\Delta}_{oracle}$, in which the variables which are heterogeneous under the null are partialled out. We expect this test to perform better than the standard delta test.

5.2 Results

In the following we focus on Specification 1 - 4, 5 and 9. All tables can be found in the Appendix A. For all simulations, the number of cross-sectional units and time periods is varied between 20 and 200. The main focus is on the Size and the Power of the test. We present the size as the rejection frequencies in percent if the hypothesis is true. That is, the number of times the delta test falsely rejects the hypothesis of homogeneous slope coefficients. The panel containing the Power of the test displays the rejection frequencies if the hypothesis is false, meaning when the true coefficients are heterogeneous. The size and test are evaluated at a level of 5%.

[TABLE 2 ABOUT HERE]

Table 2 shows the simulation results without cross-sectional dependence $\rho_f = 0$ and no serial correlated errors $\rho_u = 0$. For combinations with a small number of cross-sectional units (N) and time periods (T), the standard delta test is slightly undersized. With an increase in particular of N, the size is close to 5%. We further observe that the small sample adjusted delta test statistic performs better in small samples.

The Power of the standard delta test performs correspondingly to the size. For combinations of N and T with N and/or T being small, the power is lower. For large N and T the power reaches 100%. In particular an increase in the number of periods T leads to a better power. Our results for the power and size confirm the findings in Pesaran and Yamagata (2008).

The HAC robust version of the delta test shows a severe downward bias in the size. The size is never anywhere near its nominal value of 5%. The test also lacks power, especially in small samples.

The cross-sectional dependence robust version of the delta test is expected to perform similar to the standard delta test. This is confirmed by the results in Table 2, however the power of the test is smaller, in particular in smaller samples. A possible reason for this behaviour might be that the cross-sectional averages which are meant to take out the cross-sectional dependence, "homogenise" the coefficients. It will be interesting to see how the test will perform in a setting with cross-sectional dependence.

[TABLE 3 ABOUT HERE]

Table 3 shows results with serially correlated errors $\rho_u = 0.7$. The standard test performs badly in terms of the size, reaching almost under 100% for all combinations of N and T . The cross-sectional robust version of the test performs somewhat better, however it is still over sized. This result underlines the importance of a serial correlation robust version of the delta test.

The HAC robust delta test out performs the other two tests. However it is slightly undersized, even the small sample adjusted test never reaches more than 4%. The equivalence holds for the power of the test. The test is lacks power in small samples, especially when the number of time periods is small. However, in comparison to the two non robust tests, the HAC robust test performs superior. This finding is in line with the simulation results in Blomquist and Westerlund (2013). In their simulations the HAC robust test performs best in panels with serial correlation. Therefore it is strongly encouraged to apply this test, despite its short comings in size.

[TABLE 4 ABOUT HERE]

Table 4 compares four different combinations of kernels for the HAC consistent test. Using the bartlett kernel leads to oversized test statistics. The truncated kernel suffers from an oversize for small N and large T panels, but for large N and T panels the size comes close to its nominal value. As found in Blomquist and Westerlund (2013) prewhitening leads to much better results for the QS kernel. Once again, the test lacks power in small samples. In general the results strongly suggest to use the QS kernel in combination with prewhitening.

[TABLE 5 ABOUT HERE]

In Table 5 cross-sectional dependence is added. The standard delta test and the cross-sectional dependence robust delta test behave similar for all combination of N and T. There are two potential reasons for this finding. First of all, the cross-sectional averages might take some of the heterogeneous variation out. A second reason is that the bias of the pooled and mean group estimator is of a similar magnitude and direction. This applies to the infeasible standard OLS estimator of which $\tilde{\Delta}$ is build on as well as to the CCE type estimators $\tilde{\Delta}_{CSA}$ is based on as shown in (Pesaran 2006, Table I). This results highlights that the differences of the two tests might come at low cost and gain. However the correct method to be applies for datasets with cross-sectional dependence is the $\tilde{\Delta}_{CSA}$ test. We have not outlined the proof for the latter finding. Surprisingly the $\tilde{\Delta}_{(QS)+Prewhitened}$ is marginally outperformed by the $\tilde{\Delta}_{CSA}$, which is slightly closer to the nominal value of 5%.

[TABLE 6 ABOUT HERE]

The DGP in Table 6 contains cross-sectional dependence and serially correlated errors. As in the case with serial correlation, the standard delta test and its cross-sectional dependence robust counterpart are oversized. The HAC robust delta test performs suprsingly well, but is slightly undersized. To encompass this, we employ an additional testing procedure, $\Delta_{HAC+CSA}$ which first takes out strong cross-sectional dependence by partialling out the cross-sectional averages and then uses the heterosekdasticity autocorrelation robust delta test. While this test is oversized, the power of the test is much better than the one of the HAC robust delta test.

Tables 7 - 11 contain simulation results with 4 regressors. In general the results are similar to those with only one regressor. However the power of the tests is below those with only a single regressor.[3] Important to note is that we use the non-prewhitened version of the HAC estimator which performs better than the prewhitened equivalent. This finding is in line with Blomquist and Westerlund (2013).

[TABLE 11 ABOUT HERE]

As a final exercise we present results with 4 regressors of which one is heterogeneous $h = 1$ uner the null in Table 11. Both, the standard delta test and the cross-sectional dependence robust have a size above their nominal value, however in most cases well below 10%. The result is expected as it is harder for the test to identify the correct heterogeneous slope coefficients. This translates into a lower power. However for large combinations of N and T the size and power are in acceptable regions around 5%, respectively around 90%. The oracle test, which partials out the correct variable performs reasonably well. This implies that if a variable is know to have a heterogeneous slope parameter, partialling it out works well.

In general the simulations confirm results established in the literature for the standard and heteroskedsticity autocorrelation robust delta test. The extension which takes out cross-sectional dependence works well and can be used if cross-sectional dependence is suspected.

6 Conclusion

This paper introduces and discusses `xthst` a community contributed program for Stata. `xthst` implements tests for slope homogeneity in panels with a large number of periods over time (T) and cross-sectional units (N). Three different tests are considered. First, the standard delta test following Pesaran and Yamagata (2008), a HAC robust version following Blomquist and Westerlund (2013) and a cross-sectional dependence robust version. `xthst` supports different kernel estimators for heteroskedastic and autocorrelation robust variance estimator. The bandwidth can be chosen by hand or by a data driven method. We give several examples testing for slope homogeneity in a Solow style growth model. We show that all three test behave as expected using a Monte Carlo simulation. While the Monte Carlo results for the cross-sectional dependence robust test are promising and show that the method works, the formal derivation of a test is left for further research.

3. Results with $T = 20$ for Δ_{CSA} only contain the contemporanous value of the cross-sectional averages.

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A Appendix

A.1 Monte Carlo Simulation ($k = 1$)

	Size (%)					Power (%)				
N , T	20	50	100	150	200	20	50	100	150	200
$\tilde{\Delta}$										
20	3.20	3.45	3.10	3.15	2.95	17.30	75.35	80.70	98.60	97.75
	(5.05)	(3.70)	(3.30)	(3.30)	(3.10)	(20.05)	(76.15)	(80.80)	(98.65)	(97.75)
50	2.60	3.65	4.45	4.65	4.10	34.05	97.10	99.65	100.00	100.00
	(3.70)	(4.30)	(4.80)	(4.90)	(4.25)	(38.55)	(97.25)	(99.65)	(100.00)	(100.00)
100	3.20	4.05	5.05	4.80	4.15	87.60	100.00	100.00	100.00	100.00
	(5.15)	(4.75)	(5.50)	(5.20)	(4.30)	(89.75)	(100.00)	(100.00)	(100.00)	(100.00)
150	3.20	4.40	5.20	4.20	4.85	96.15	100.00	100.00	100.00	100.00
	(4.65)	(5.00)	(5.70)	(4.40)	(5.10)	(97.10)	(100.00)	(100.00)	(100.00)	(100.00)
200	3.45	3.40	4.10	4.15	4.70	99.85	100.00	100.00	100.00	100.00
	(4.95)	(4.30)	(4.40)	(4.50)	(4.80)	(99.85)	(100.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{HAC(QS)+Prewhitened}$										
20	1.70	1.80	2.35	2.90	2.15	3.80	27.10	68.35	96.00	96.45
	(2.55)	(2.40)	(2.50)	(2.95)	(2.20)	(5.75)	(28.85)	(68.75)	(96.25)	(96.55)
50	1.65	2.15	3.50	4.15	3.35	8.65	69.30	94.65	99.95	100.00
	(2.60)	(2.45)	(3.75)	(4.30)	(3.60)	(11.40)	(70.80)	(94.80)	(99.95)	(100.00)
100	1.65	2.80	3.85	3.90	3.85	23.90	99.40	100.00	100.00	100.00
	(2.70)	(3.15)	(4.35)	(4.10)	(3.90)	(29.60)	(99.45)	(100.00)	(100.00)	(100.00)
150	1.95	3.40	4.15	3.15	4.15	42.30	99.65	100.00	100.00	100.00
	(3.05)	(3.95)	(4.45)	(3.25)	(4.30)	(49.10)	(99.70)	(100.00)	(100.00)	(100.00)
200	1.40	2.20	2.90	3.05	4.25	54.60	100.00	100.00	100.00	100.00
	(2.60)	(2.45)	(3.10)	(3.15)	(4.30)	(61.25)	(100.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{CSA}$										
20	2.65	3.30	3.40	3.00	3.25	12.40	61.35	69.25	96.25	95.85
	(4.25)	(4.20)	(3.85)	(3.35)	(3.40)	(15.10)	(62.75)	(69.95)	(96.35)	(95.85)
50	2.55	3.45	3.60	4.55	4.30	23.55	90.40	99.05	100.00	100.00
	(4.35)	(4.40)	(4.20)	(4.60)	(4.50)	(28.55)	(91.25)	(99.05)	(100.00)	(100.00)
100	2.95	4.10	4.40	5.05	4.35	79.70	100.00	100.00	100.00	100.00
	(5.40)	(5.10)	(4.70)	(5.25)	(4.75)	(83.65)	(100.00)	(100.00)	(100.00)	(100.00)
150	2.70	4.00	4.65	3.50	4.85	90.75	100.00	100.00	100.00	100.00
	(4.10)	(4.90)	(5.15)	(3.65)	(4.95)	(92.75)	(100.00)	(100.00)	(100.00)	(100.00)
200	2.80	2.90	3.50	4.80	4.40	98.95	100.00	100.00	100.00	100.00
	(4.55)	(3.75)	(4.00)	(5.05)	(4.50)	(99.30)	(100.00)	(100.00)	(100.00)	(100.00)

Table 2: Specification 1 - Size and Power of Delta test with no cross-sectional dependence and heteroskedastic normal iid errors.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 1 exogenous regressor. For a definition of the DGP see Section 5.

N , T	Size (%)					Power (%)				
	20	50	100	150	200	20	50	100	150	200
$\tilde{\Delta}$										
20	19.30	31.25	35.10	31.00	23.50	75.65	88.15	95.80	99.90	98.55
	(22.90)	(32.80)	(35.70)	(31.50)	(23.85)	(78.25)	(88.85)	(95.90)	(99.90)	(98.60)
50	30.00	46.30	49.65	48.10	60.40	96.95	99.55	100.00	99.70	100.00
	(35.10)	(47.90)	(50.40)	(48.65)	(60.60)	(97.70)	(99.55)	(100.00)	(99.70)	(100.00)
100	58.55	74.40	78.80	86.90	80.45	99.35	100.00	100.00	100.00	100.00
	(63.10)	(75.95)	(79.30)	(87.25)	(80.70)	(99.60)	(100.00)	(100.00)	(100.00)	(100.00)
150	72.55	88.60	94.05	91.50	88.80	99.95	100.00	100.00	100.00	100.00
	(76.35)	(89.60)	(94.20)	(91.65)	(89.00)	(99.95)	(100.00)	(100.00)	(100.00)	(100.00)
200	84.45	95.25	97.50	97.30	97.15	100.00	100.00	100.00	100.00	100.00
	(87.05)	(95.40)	(97.50)	(97.50)	(97.25)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{HAC(QS)+Prewhitened}$										
20	1.90	1.80	1.85	1.50	2.50	7.65	17.00	51.60	99.45	91.20
	(2.55)	(2.10)	(1.95)	(1.75)	(2.60)	(9.40)	(18.50)	(52.10)	(99.45)	(91.30)
50	2.05	1.95	2.60	2.50	3.35	23.15	72.30	92.05	92.10	99.70
	(3.80)	(2.30)	(2.80)	(2.60)	(3.50)	(27.60)	(74.15)	(92.40)	(92.25)	(99.70)
100	1.85	2.00	3.65	3.45	3.45	23.20	95.00	100.00	100.00	100.00
	(2.70)	(2.65)	(3.85)	(3.75)	(3.55)	(28.55)	(95.80)	(100.00)	(100.00)	(100.00)
150	2.50	2.05	3.05	3.55	3.30	33.25	99.30	100.00	100.00	100.00
	(3.85)	(2.65)	(3.45)	(3.70)	(3.45)	(38.85)	(99.50)	(100.00)	(100.00)	(100.00)
200	2.40	1.95	3.65	3.55	3.95	55.35	95.15	100.00	100.00	100.00
	(3.65)	(2.45)	(3.95)	(3.60)	(4.00)	(60.70)	(95.55)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{CSA}$										
20	14.55	26.65	32.55	30.15	21.95	63.95	78.45	92.90	99.95	97.70
	(18.55)	(28.10)	(33.35)	(30.70)	(22.15)	(68.05)	(79.30)	(92.95)	(99.95)	(97.75)
50	23.50	40.70	46.10	46.70	57.25	93.90	99.00	99.75	99.55	100.00
	(28.55)	(42.90)	(47.00)	(47.55)	(57.45)	(95.20)	(99.20)	(99.75)	(99.55)	(100.00)
100	45.70	68.20	75.10	85.25	79.75	98.55	100.00	100.00	100.00	100.00
	(51.80)	(70.90)	(75.80)	(85.55)	(80.00)	(99.00)	(100.00)	(100.00)	(100.00)	(100.00)
150	60.10	81.55	93.25	90.10	86.80	99.75	100.00	100.00	100.00	100.00
	(65.00)	(83.00)	(93.55)	(90.50)	(87.05)	(99.75)	(100.00)	(100.00)	(100.00)	(100.00)
200	71.90	91.45	96.55	96.80	96.45	100.00	100.00	100.00	100.00	100.00
	(75.55)	(92.30)	(96.75)	(96.95)	(96.55)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)

Table 3: Specification 2 - Size and Power of Delta test with no cross-sectional dependence and serially correlated errors with heteroskedasticity.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 1 exogenous regressor. For a definition of the DGP see Section 5.

	Size (%)					Power (%)				
N , T	20	50	100	150	200	20	50	100	150	200
$\tilde{\Delta}_{HAC(bartlett)}$										
20	20.05	30.75	34.70	31.00	23.20	64.35	84.50	94.90	100.00	98.40
	(23.85)	(32.35)	(35.15)	(31.60)	(23.50)	(68.70)	(85.15)	(94.90)	(100.00)	(98.40)
50	33.50	48.00	49.55	47.45	60.15	93.15	99.35	100.00	99.60	100.00
	(38.95)	(49.30)	(50.55)	(47.90)	(60.45)	(94.55)	(99.40)	(100.00)	(99.65)	(100.00)
100	67.35	75.65	78.90	87.35	80.75	98.85	100.00	100.00	100.00	100.00
	(71.30)	(77.15)	(79.80)	(87.75)	(81.25)	(99.05)	(100.00)	(100.00)	(100.00)	(100.00)
150	81.05	90.30	94.35	91.95	88.85	99.85	100.00	100.00	100.00	100.00
	(85.40)	(90.95)	(94.45)	(92.05)	(89.45)	(99.85)	(100.00)	(100.00)	(100.00)	(100.00)
200	90.40	96.25	97.90	97.45	97.25	100.00	100.00	100.00	100.00	100.00
	(92.95)	(96.70)	(98.00)	(97.50)	(97.30)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{HAC(QS)}$										
20	1.00	2.25	2.65	3.35	4.60	6.25	18.95	43.25	99.60	92.15
	(1.55)	(2.85)	(3.10)	(3.55)	(4.75)	(7.95)	(20.50)	(44.10)	(99.60)	(92.30)
50	1.20	3.05	4.55	4.60	5.60	23.50	82.75	94.40	94.60	99.70
	(2.10)	(3.60)	(4.85)	(4.70)	(5.85)	(28.95)	(84.00)	(94.60)	(94.70)	(99.70)
100	0.95	4.00	5.90	6.85	7.20	24.25	98.25	100.00	100.00	100.00
	(1.90)	(4.90)	(6.50)	(7.05)	(7.50)	(30.50)	(98.50)	(100.00)	(100.00)	(100.00)
150	0.75	4.05	8.20	7.50	9.10	32.60	99.80	100.00	100.00	100.00
	(1.65)	(4.75)	(8.70)	(7.85)	(9.45)	(38.65)	(99.80)	(100.00)	(100.00)	(100.00)
200	1.30	5.45	9.70	9.30	9.35	58.85	98.75	100.00	100.00	100.00
	(2.45)	(6.05)	(10.00)	(9.75)	(9.70)	(65.65)	(99.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{HAC(QS)+Whitening}$										
20	1.90	1.80	1.85	1.50	2.50	7.65	17.00	51.60	99.45	91.20
	(2.55)	(2.10)	(1.95)	(1.75)	(2.60)	(9.40)	(18.50)	(52.10)	(99.45)	(91.30)
50	2.05	1.95	2.60	2.50	3.35	23.15	72.30	92.05	92.10	99.70
	(3.80)	(2.30)	(2.80)	(2.60)	(3.50)	(27.60)	(74.15)	(92.40)	(92.25)	(99.70)
100	1.85	2.00	3.65	3.45	3.45	23.20	95.00	100.00	100.00	100.00
	(2.70)	(2.65)	(3.85)	(3.75)	(3.55)	(28.55)	(95.80)	(100.00)	(100.00)	(100.00)
150	2.50	2.05	3.05	3.55	3.30	33.25	99.30	100.00	100.00	100.00
	(3.85)	(2.65)	(3.45)	(3.70)	(3.45)	(38.85)	(99.50)	(100.00)	(100.00)	(100.00)
200	2.40	1.95	3.65	3.55	3.95	55.35	95.15	100.00	100.00	100.00
	(3.65)	(2.45)	(3.95)	(3.60)	(4.00)	(60.70)	(95.55)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{HAC(truncated)}$										
20	5.30	3.60	2.30	1.70	2.70	12.00	20.15	46.85	98.65	88.40
	(6.60)	(4.00)	(2.40)	(1.95)	(2.85)	(15.10)	(21.45)	(47.65)	(98.65)	(88.70)
50	11.75	5.95	3.65	2.40	3.70	37.15	67.80	85.35	87.15	99.55
	(13.65)	(6.60)	(3.75)	(2.55)	(3.85)	(43.35)	(70.20)	(85.90)	(87.35)	(99.55)
100	20.10	9.45	5.40	3.70	3.35	53.50	95.90	99.95	100.00	100.00
	(23.00)	(10.25)	(5.65)	(3.85)	(3.80)	(59.90)	(96.70)	(99.95)	(100.00)	(100.00)
150	30.10	12.55	6.50	3.60	4.05	71.20	99.35	100.00	100.00	100.00
	(34.35)	(13.60)	(6.80)	(3.80)	(4.20)	(76.75)	(99.55)	(100.00)	(100.00)	(100.00)
200	38.80	14.05	8.10	4.30	3.35	88.45	97.75	100.00	100.00	100.00
	(43.70)	(15.70)	(8.50)	(4.45)	(3.50)	(91.35)	(98.05)	(100.00)	(100.00)	(100.00)

Table 4: Specification 2 - Size and Power of Delta test with no cross-sectional dependence and serially correlated errors with heteroskedasticity.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 1 exogenous regressor. For a definition of the DGP see Section 5.

N , T	Size (%) 20	50	100	150	200	Power (%) 20	50	100	150	200
$\tilde{\Delta}$										
20	3.35	3.95	3.60	3.65	3.95	8.15	50.85	65.10	97.35	91.45
	(5.40)	(4.20)	(4.05)	(3.90)	(4.10)	(10.25)	(52.25)	(65.70)	(97.35)	(91.55)
50	2.75	4.45	3.30	3.40	3.95	59.90	97.50	99.05	100.00	100.00
	(3.95)	(5.40)	(3.55)	(3.40)	(4.15)	(64.20)	(97.55)	(99.05)	(100.00)	(100.00)
100	3.05	4.20	3.80	4.55	4.40	95.20	100.00	100.00	100.00	100.00
	(4.90)	(4.80)	(4.30)	(4.90)	(4.50)	(96.45)	(100.00)	(100.00)	(100.00)	(100.00)
150	3.50	3.65	5.05	5.15	4.05	86.70	100.00	100.00	100.00	100.00
	(4.60)	(4.35)	(5.35)	(5.40)	(4.35)	(89.50)	(100.00)	(100.00)	(100.00)	(100.00)
200	2.80	4.40	4.40	4.65	4.45	90.40	100.00	100.00	100.00	100.00
	(4.70)	(4.95)	(4.70)	(5.00)	(4.80)	(92.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{HAC(QS)+Prewhitened}$										
20	2.25	1.75	2.45	3.00	3.00	3.70	24.70	46.10	95.25	85.45
	(2.90)	(2.15)	(2.55)	(3.30)	(3.05)	(5.00)	(26.25)	(47.10)	(95.45)	(85.65)
50	1.65	3.15	2.80	2.50	3.65	12.25	77.60	95.45	100.00	100.00
	(2.50)	(3.60)	(3.05)	(2.65)	(3.80)	(15.85)	(79.30)	(95.55)	(100.00)	(100.00)
100	2.60	2.20	3.00	3.75	3.20	31.60	97.90	100.00	100.00	100.00
	(3.60)	(2.65)	(3.20)	(3.95)	(3.45)	(37.90)	(98.30)	(100.00)	(100.00)	(100.00)
150	1.30	2.15	3.95	3.65	3.75	24.60	100.00	100.00	100.00	100.00
	(2.40)	(2.80)	(4.15)	(4.00)	(3.85)	(29.65)	(100.00)	(100.00)	(100.00)	(100.00)
200	2.05	2.40	3.65	3.75	3.35	31.10	99.90	100.00	100.00	100.00
	(2.90)	(2.95)	(4.05)	(3.85)	(3.55)	(37.75)	(99.90)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{CSA}$										
20	3.05	3.35	3.65	3.60	3.85	5.75	30.30	53.65	94.20	86.45
	(5.30)	(4.05)	(3.85)	(3.80)	(3.95)	(8.05)	(32.00)	(54.50)	(94.20)	(86.50)
50	3.00	3.65	3.35	3.30	4.20	49.70	92.45	98.15	100.00	100.00
	(4.60)	(4.45)	(3.75)	(3.50)	(4.40)	(54.90)	(93.60)	(98.20)	(100.00)	(100.00)
100	3.20	3.85	4.20	4.45	4.55	90.85	100.00	100.00	100.00	100.00
	(5.05)	(5.05)	(4.50)	(4.65)	(4.75)	(93.20)	(100.00)	(100.00)	(100.00)	(100.00)
150	2.95	4.15	4.70	4.95	3.95	77.20	100.00	100.00	100.00	100.00
	(4.70)	(5.00)	(5.20)	(5.20)	(4.40)	(82.05)	(100.00)	(100.00)	(100.00)	(100.00)
200	2.90	4.35	4.95	4.35	4.05	82.00	100.00	100.00	100.00	100.00
	(4.10)	(5.15)	(5.20)	(4.50)	(4.25)	(86.30)	(100.00)	(100.00)	(100.00)	(100.00)

Table 5: Specification 3 - Size and Power of Delta test with cross-sectional dependence and heteroskedastic normal iid errors.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 1 exogenous regressor. For a definition of the DGP see Section 5.

	Size (%)					Power (%)				
N , T	20	50	100	150	200	20	50	100	150	200
$\tilde{\Delta}$										
20	18.20	27.00	26.90	40.40	29.70	46.55	81.95	87.70	98.50	99.85
	(21.80)	(28.35)	(27.55)	(40.90)	(30.15)	(50.80)	(83.35)	(87.95)	(98.60)	(99.85)
50	28.25	52.00	42.85	59.80	58.05	92.05	99.90	98.80	100.00	100.00
	(33.20)	(53.30)	(43.65)	(60.45)	(58.60)	(93.45)	(99.90)	(98.80)	(100.00)	(100.00)
100	53.85	82.50	77.15	85.50	85.10	99.85	100.00	100.00	100.00	100.00
	(59.85)	(83.40)	(77.60)	(86.00)	(85.40)	(99.90)	(100.00)	(100.00)	(100.00)	(100.00)
150	72.55	86.05	93.55	91.30	95.15	100.00	100.00	100.00	100.00	100.00
	(76.25)	(87.25)	(93.80)	(91.60)	(95.25)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
200	82.30	96.75	96.65	95.45	98.00	100.00	100.00	100.00	100.00	100.00
	(85.60)	(97.00)	(96.75)	(95.70)	(98.05)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{HAC(QS)+Prewhitened}$										
20	1.55	1.35	2.25	2.95	2.60	3.55	21.65	41.15	85.60	98.05
	(2.55)	(1.50)	(2.35)	(3.00)	(2.75)	(4.35)	(23.25)	(41.60)	(85.65)	(98.05)
50	1.70	2.90	2.75	2.85	2.90	14.65	59.50	66.95	99.75	100.00
	(2.60)	(3.35)	(3.10)	(2.90)	(3.00)	(18.10)	(61.70)	(67.75)	(99.75)	(100.00)
100	2.30	2.45	2.05	2.85	3.55	25.15	74.10	99.50	99.25	100.00
	(3.65)	(2.65)	(2.55)	(3.10)	(3.55)	(30.65)	(76.15)	(99.50)	(99.30)	(100.00)
150	2.10	1.75	3.65	3.10	3.40	40.50	98.85	100.00	100.00	100.00
	(2.90)	(2.05)	(3.85)	(3.30)	(3.55)	(47.60)	(99.10)	(100.00)	(100.00)	(100.00)
200	1.65	1.35	2.45	3.05	3.15	45.30	98.95	100.00	100.00	100.00
	(2.95)	(1.60)	(2.60)	(3.30)	(3.15)	(51.90)	(99.15)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{CSA}$										
20	13.50	22.90	26.55	39.35	28.85	34.35	69.65	81.15	97.80	99.50
	(17.20)	(24.30)	(27.40)	(40.00)	(29.00)	(38.40)	(70.80)	(81.65)	(97.90)	(99.55)
50	22.00	44.05	40.95	58.35	55.55	86.05	99.65	97.65	100.00	100.00
	(25.85)	(46.10)	(41.85)	(59.10)	(55.95)	(88.45)	(99.65)	(97.70)	(100.00)	(100.00)
100	42.25	73.55	73.85	83.85	84.25	98.90	100.00	100.00	100.00	100.00
	(47.90)	(75.05)	(74.40)	(84.50)	(84.45)	(99.30)	(100.00)	(100.00)	(100.00)	(100.00)
150	59.85	78.90	91.05	90.90	94.95	99.95	100.00	100.00	100.00	100.00
	(66.10)	(80.85)	(91.65)	(91.15)	(95.05)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
200	69.60	93.40	95.45	94.50	97.55	100.00	100.00	100.00	100.00	100.00
	(74.25)	(94.15)	(95.55)	(94.60)	(97.65)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{CSA+HAC(QS)+Prewhitened}$										
20	4.10	6.45	5.35	4.60	4.55	8.25	33.80	45.20	86.10	97.65
	(5.65)	(7.40)	(5.65)	(4.75)	(4.80)	(11.10)	(36.25)	(46.25)	(86.25)	(97.70)
50	6.35	13.55	7.85	7.20	5.20	33.35	80.40	76.50	99.75	100.00
	(8.05)	(15.20)	(8.15)	(7.30)	(5.25)	(40.35)	(82.30)	(77.45)	(99.75)	(100.00)
100	12.90	24.95	11.65	9.45	6.85	57.10	95.15	99.90	99.80	100.00
	(16.65)	(27.10)	(12.30)	(9.65)	(6.90)	(62.80)	(95.85)	(99.90)	(99.80)	(100.00)
150	20.00	32.75	14.95	10.40	8.65	77.40	99.90	100.00	100.00	100.00
	(25.10)	(35.05)	(15.95)	(10.70)	(8.70)	(82.25)	(99.95)	(100.00)	(100.00)	(100.00)
200	25.75	45.40	18.25	12.85	8.60	85.00	100.00	100.00	100.00	100.00
	(30.75)	(48.45)	(19.35)	(13.20)	(9.05)	(88.20)	(100.00)	(100.00)	(100.00)	(100.00)

Table 6: Specification 4 - Size and Power of Delta test with cross-sectional dependence and serially correlated errors with heteroskedasticity.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 1 exogenous regressor. For a definition of the DGP see Section 5.

A.2 Monte Carlo Simulation ($k = 4$)

	Size (%)					Power (%)				
N , T	20	50	100	150	200	20	50	100	150	200
$\tilde{\Delta}$										
20	2.10	4.30	4.95	4.35	4.50	1.95	4.05	7.40	7.15	6.60
	(5.90)	(5.95)	(5.75)	(4.80)	(4.90)	(5.35)	(5.30)	(8.10)	(7.70)	(6.90)
50	2.35	3.80	4.05	4.90	4.20	2.00	3.85	15.80	23.20	10.95
	(4.90)	(5.60)	(4.80)	(5.40)	(4.60)	(5.05)	(5.30)	(17.15)	(24.05)	(11.25)
100	2.55	3.35	4.40	4.00	4.35	2.45	11.15	26.00	29.15	67.05
	(6.40)	(4.65)	(5.20)	(4.35)	(4.85)	(6.30)	(13.85)	(27.50)	(30.10)	(67.70)
150	1.40	3.40	4.25	4.60	5.80	1.25	6.80	70.10	51.00	93.55
	(4.95)	(5.40)	(4.90)	(5.05)	(6.00)	(4.50)	(8.15)	(71.65)	(52.00)	(93.75)
200	2.75	3.55	4.35	3.85	5.00	3.60	25.05	94.60	53.90	97.50
	(6.35)	(4.80)	(5.20)	(4.30)	(5.40)	(7.75)	(28.75)	(95.20)	(55.10)	(97.55)
$\tilde{\Delta}_{HAC(QS)}$										
20	1.95	3.10	3.85	3.80	3.90	2.05	2.75	3.80	4.75	4.80
	(6.65)	(4.20)	(4.30)	(4.20)	(4.45)	(6.30)	(3.85)	(4.50)	(5.40)	(4.95)
50	3.60	2.75	3.20	4.20	4.30	3.35	2.25	4.95	6.60	7.35
	(10.30)	(3.85)	(3.60)	(4.55)	(4.50)	(9.05)	(3.00)	(5.55)	(7.30)	(7.70)
100	8.80	3.35	3.75	4.10	3.80	8.20	1.70	10.25	15.70	39.60
	(20.60)	(5.35)	(4.25)	(4.55)	(4.15)	(18.20)	(2.75)	(11.30)	(16.30)	(40.55)
150	14.15	5.10	3.85	4.00	4.55	11.85	2.25	20.95	18.95	74.10
	(30.85)	(6.65)	(4.55)	(4.60)	(4.90)	(26.15)	(3.35)	(22.30)	(20.05)	(74.70)
200	21.25	5.25	4.90	3.95	4.65	16.85	1.85	37.45	25.45	82.65
	(38.95)	(7.20)	(5.65)	(4.65)	(4.90)	(34.25)	(2.80)	(39.45)	(26.95)	(83.25)
$\tilde{\Delta}_{CSA}$										
20	1.85	2.85	4.65	4.25	5.00	1.90	2.95	4.95	5.35	5.30
	(7.20)	(5.65)	(5.85)	(4.90)	(5.60)	(6.75)	(5.25)	(6.30)	(6.00)	(5.80)
50	1.55	2.55	3.50	4.75	3.90	1.55	2.90	7.00	13.80	7.65
	(5.75)	(5.15)	(4.20)	(5.30)	(4.40)	(5.25)	(5.45)	(8.30)	(14.40)	(8.40)
100	1.70	2.85	4.10	4.35	4.95	1.65	3.20	13.00	17.80	50.75
	(6.15)	(4.75)	(4.70)	(5.05)	(5.40)	(6.15)	(6.80)	(14.75)	(19.20)	(51.90)
150	1.75	2.85	4.05	4.00	4.65	1.45	2.75	41.60	31.05	86.85
	(6.20)	(5.70)	(5.25)	(4.50)	(4.95)	(5.70)	(5.40)	(45.10)	(32.35)	(87.50)
200	1.65	2.80	3.50	5.00	5.40	1.60	5.55	75.10	31.95	91.25
	(6.45)	(5.50)	(4.35)	(5.75)	(5.65)	(6.30)	(9.55)	(77.35)	(33.40)	(91.60)

Table 7: Specification 5 - Size and Power of Delta test with no cross-sectional dependence and heteroskedastic normal iid errors.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 4 exogenous regressors. For a definition of the DGP see Section 5.

N , T	Size (%) 20	50	100	150	200	Power (%) 20	50	100	150	200
$\tilde{\Delta}$										
20	22.70	43.35	62.20	60.60	65.30	24.20	48.20	68.70	76.55	76.20
	(31.95)	(46.90)	(63.50)	(61.15)	(66.05)	(34.20)	(51.80)	(69.85)	(77.30)	(76.90)
50	60.85	92.15	95.90	94.90	95.80	66.65	94.55	97.65	98.35	99.70
	(72.40)	(93.65)	(96.35)	(95.35)	(95.90)	(77.05)	(96.00)	(98.00)	(98.40)	(99.70)
100	91.00	99.55	99.70	99.95	99.70	94.05	99.85	100.00	100.00	100.00
	(95.75)	(99.70)	(99.80)	(99.95)	(99.70)	(97.15)	(99.85)	(100.00)	(100.00)	(100.00)
150	97.90	100.00	100.00	100.00	100.00	98.55	100.00	100.00	100.00	100.00
	(98.90)	(100.00)	(100.00)	(100.00)	(100.00)	(99.50)	(100.00)	(100.00)	(100.00)	(100.00)
200	99.75	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00
	(99.95)	(100.00)	(100.00)	(100.00)	(100.00)	(99.95)	(100.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{HAC(QS)}$										
20	1.00	1.20	1.95	2.40	3.60	1.10	1.10	2.20	6.55	6.25
	(4.25)	(1.85)	(2.25)	(2.55)	(3.95)	(4.10)	(1.50)	(2.40)	(6.90)	(6.65)
50	1.30	1.70	2.35	7.40	4.65	1.35	1.90	3.60	14.85	32.00
	(5.40)	(2.20)	(2.90)	(8.05)	(5.00)	(5.25)	(2.35)	(4.20)	(15.85)	(32.80)
100	4.95	0.65	3.55	6.20	12.80	4.35	1.30	10.90	38.55	53.65
	(14.50)	(1.25)	(4.05)	(6.80)	(13.50)	(13.15)	(1.85)	(12.05)	(40.10)	(54.70)
150	10.40	0.90	7.95	11.25	12.30	8.40	2.25	19.20	39.40	84.30
	(24.30)	(1.70)	(8.90)	(11.90)	(13.05)	(22.05)	(3.25)	(20.65)	(40.85)	(84.75)
200	21.30	1.40	8.40	13.60	15.40	17.70	5.25	44.40	84.00	89.15
	(40.70)	(2.10)	(9.85)	(14.35)	(16.55)	(36.90)	(7.35)	(46.60)	(84.80)	(89.75)
$\tilde{\Delta}_{CSA}$										
20	5.50	17.50	53.10	52.55	61.95	5.55	19.60	57.45	66.45	71.45
	(13.90)	(23.65)	(55.35)	(53.65)	(62.70)	(14.55)	(26.05)	(59.30)	(68.10)	(72.20)
50	17.85	59.15	91.50	92.50	94.40	20.25	63.25	94.00	96.55	99.30
	(32.55)	(67.20)	(92.65)	(93.15)	(94.75)	(36.70)	(71.50)	(94.60)	(96.70)	(99.35)
100	39.70	86.85	99.65	99.70	99.80	45.15	91.65	99.80	100.00	100.00
	(61.00)	(91.30)	(99.65)	(99.75)	(99.85)	(65.75)	(94.35)	(99.90)	(100.00)	(100.00)
150	56.40	97.20	100.00	100.00	100.00	63.45	98.90	100.00	100.00	100.00
	(73.40)	(98.65)	(100.00)	(100.00)	(100.00)	(79.25)	(99.35)	(100.00)	(100.00)	(100.00)
200	73.85	99.35	100.00	100.00	100.00	80.05	99.95	100.00	100.00	100.00
	(88.10)	(99.55)	(100.00)	(100.00)	(100.00)	(91.20)	(100.00)	(100.00)	(100.00)	(100.00)

Table 8: Specification 6 - Size and Power of Delta test with no cross-sectional dependence and serially correlated errors with heteroskedasticity.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 4 exogenous regressors. For a definition of the DGP see Section 5.

	Size (%)					Power (%)				
N , T	20	50	100	150	200	20	50	100	150	200
$\tilde{\Delta}$										
20	1.85	4.60	4.65	5.55	4.40	1.70	5.10	8.95	5.10	8.60
	(5.60)	(6.40)	(5.55)	(5.95)	(4.60)	(5.05)	(6.30)	(9.95)	(5.35)	(8.80)
50	2.00	4.15	5.05	4.35	4.75	1.85	5.60	35.25	27.75	31.00
	(5.75)	(5.65)	(5.65)	(4.95)	(5.15)	(5.00)	(6.65)	(37.25)	(28.90)	(32.00)
100	2.75	4.20	4.25	4.05	4.00	2.40	4.35	16.75	42.80	80.90
	(6.30)	(5.55)	(5.05)	(4.75)	(4.45)	(5.90)	(5.95)	(17.90)	(44.40)	(81.45)
150	1.95	3.80	4.70	5.00	5.25	1.80	9.45	32.60	67.15	65.45
	(5.60)	(5.05)	(5.40)	(5.25)	(5.45)	(5.10)	(11.80)	(34.35)	(68.10)	(66.30)
200	2.30	3.55	4.25	4.70	4.60	2.75	9.15	35.10	84.70	98.60
	(5.65)	(5.00)	(4.85)	(5.15)	(4.85)	(5.80)	(11.35)	(37.30)	(85.25)	(98.65)
$\tilde{\Delta}_{HAC(QS)}$										
20	1.75	3.55	3.90	4.45	3.60	1.55	2.55	3.80	4.00	5.35
	(5.70)	(4.85)	(4.55)	(5.00)	(3.85)	(5.60)	(3.70)	(4.50)	(4.45)	(5.60)
50	3.80	3.50	3.90	3.85	4.60	3.75	1.95	9.70	14.45	19.05
	(11.20)	(5.00)	(4.95)	(3.95)	(5.05)	(10.70)	(3.30)	(10.65)	(15.10)	(19.50)
100	8.80	3.95	3.70	3.50	4.10	7.45	2.10	6.35	18.85	53.75
	(21.20)	(5.75)	(4.55)	(3.95)	(4.35)	(19.70)	(3.05)	(7.15)	(19.75)	(54.85)
150	13.50	4.55	4.65	4.60	4.95	11.60	1.85	11.15	33.25	48.15
	(28.85)	(5.90)	(5.70)	(5.20)	(5.20)	(24.15)	(2.75)	(12.20)	(34.85)	(49.15)
200	21.55	5.45	4.10	4.45	4.20	16.85	2.50	9.90	45.80	91.50
	(39.60)	(7.75)	(4.60)	(4.85)	(4.40)	(34.20)	(3.50)	(11.00)	(47.85)	(91.70)
$\tilde{\Delta}_{CSA}$										
20	1.60	2.75	3.90	4.20	5.15	1.25	2.90	5.35	4.40	6.20
	(6.30)	(5.80)	(4.70)	(4.75)	(5.80)	(5.85)	(5.50)	(6.30)	(4.85)	(6.65)
50	1.80	2.75	4.15	5.30	4.85	1.75	2.95	18.10	15.70	20.95
	(6.50)	(5.35)	(5.25)	(5.85)	(5.15)	(6.30)	(5.55)	(19.60)	(16.80)	(21.65)
100	1.45	3.25	3.85	4.80	4.30	1.25	3.20	9.60	24.80	66.70
	(5.30)	(5.65)	(4.95)	(5.35)	(5.00)	(5.25)	(5.65)	(10.90)	(26.20)	(67.35)
150	2.10	2.45	3.90	3.95	4.75	1.70	3.00	14.35	47.75	48.50
	(6.00)	(4.75)	(4.55)	(4.40)	(5.30)	(5.80)	(5.55)	(16.35)	(49.25)	(49.70)
200	1.80	2.70	4.40	4.10	4.65	1.60	3.80	16.35	65.05	94.75
	(6.30)	(5.90)	(5.25)	(4.70)	(5.10)	(5.75)	(6.00)	(18.45)	(66.60)	(95.00)

Table 9: Specification 7 - Size and Power of Delta test with cross-sectional dependence and heteroskedastic normal iid errors.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 4 exogenous regressors. For a definition of the DGP see Section 5.

N , T	Size (%)					Power (%)				
	20	50	100	150	200	20	50	100	150	200
$\tilde{\Delta}$										
20	13.65	46.30	51.35	70.25	62.10	13.80	53.05	70.85	88.50	80.75
	(21.00)	(50.10)	(52.65)	(71.05)	(62.85)	(21.00)	(56.70)	(71.85)	(89.30)	(81.35)
50	60.20	93.05	94.85	95.25	96.60	62.15	95.75	98.15	98.70	99.60
	(72.40)	(94.45)	(95.45)	(95.45)	(96.75)	(74.70)	(96.75)	(98.25)	(98.75)	(99.65)
100	89.00	99.45	99.85	100.00	99.95	90.80	99.75	100.00	100.00	100.00
	(93.50)	(99.55)	(99.85)	(100.00)	(99.95)	(94.95)	(99.85)	(100.00)	(100.00)	(100.00)
150	98.05	100.00	100.00	100.00	99.95	98.75	100.00	100.00	100.00	100.00
	(99.15)	(100.00)	(100.00)	(100.00)	(99.95)	(99.45)	(100.00)	(100.00)	(100.00)	(100.00)
200	99.85	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00
	(99.95)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{HAC(QS)}$										
20	1.55	0.70	2.10	2.95	3.25	1.45	0.65	3.25	11.35	6.35
	(4.85)	(1.25)	(2.35)	(3.05)	(3.55)	(4.95)	(1.30)	(3.75)	(12.05)	(6.65)
50	2.85	0.65	3.10	4.25	4.60	3.40	0.85	8.85	14.30	17.40
	(10.50)	(1.15)	(3.50)	(4.90)	(4.80)	(9.90)	(1.50)	(10.00)	(15.15)	(18.10)
100	6.05	0.90	4.45	8.85	9.30	5.35	1.40	15.25	34.30	58.05
	(17.15)	(1.60)	(4.90)	(9.75)	(9.70)	(15.25)	(2.30)	(16.55)	(35.55)	(59.50)
150	12.90	1.35	5.10	15.00	12.50	11.75	2.25	30.90	66.90	88.60
	(30.35)	(1.95)	(5.80)	(16.05)	(13.25)	(26.75)	(3.25)	(33.05)	(68.80)	(88.90)
200	17.55	1.30	8.50	11.15	16.90	14.40	3.60	39.85	56.15	79.50
	(33.15)	(2.00)	(9.65)	(12.05)	(17.60)	(30.25)	(5.25)	(42.50)	(57.50)	(80.75)
$\tilde{\Delta}_{CSA}$										
20	2.85	17.95	40.45	64.35	57.95	2.90	20.45	55.25	81.85	73.10
	(8.40)	(24.20)	(42.90)	(65.40)	(58.60)	(8.30)	(26.95)	(57.15)	(82.45)	(74.10)
50	17.80	61.90	88.95	92.85	94.90	19.35	65.85	93.80	96.90	98.80
	(33.15)	(70.10)	(90.45)	(93.40)	(95.00)	(35.30)	(73.10)	(94.10)	(97.20)	(98.95)
100	37.55	87.25	99.50	99.85	99.85	41.75	91.15	100.00	100.00	100.00
	(56.85)	(90.75)	(99.55)	(99.85)	(99.85)	(60.10)	(94.20)	(100.00)	(100.00)	(100.00)
150	59.00	95.95	100.00	100.00	100.00	65.25	98.05	100.00	100.00	100.00
	(75.60)	(97.65)	(100.00)	(100.00)	(100.00)	(80.35)	(98.65)	(100.00)	(100.00)	(100.00)
200	72.75	99.35	100.00	100.00	100.00	80.85	99.70	100.00	100.00	100.00
	(86.35)	(99.65)	(100.00)	(100.00)	(100.00)	(91.45)	(99.90)	(100.00)	(100.00)	(100.00)
$\tilde{\Delta}_{CSA+HAC(QS)+Whitening}$										
20	99.80	100.00	96.55	94.70	85.10	99.75	99.95	97.65	97.80	90.85
	(100.00)	(100.00)	(96.90)	(95.05)	(85.40)	(99.80)	(99.95)	(98.05)	(97.95)	(91.10)
50	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
150	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
200	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)

Table 10: Specification 8 - Size and Power of Delta test with cross-sectional dependence and serially correlated errors with heteroskedasticity.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 4 exogenous regressors. For a definition of the DGP see Section 5.

	Size (%)					Power (%)				
N , T	20	50	100	150	200	20	50	100	150	200
$\tilde{\Delta}$										
20	2.10	3.90	6.20	5.05	4.60	1.60	3.40	8.65	8.35	4.60
	(6.60)	(5.40)	(6.75)	(5.75)	(5.10)	(5.60)	(5.15)	(9.25)	(9.05)	(5.55)
50	1.50	3.60	4.30	7.60	16.90	1.55	4.85	6.45	23.00	59.20
	(4.90)	(4.85)	(4.60)	(8.30)	(17.55)	(4.40)	(6.25)	(7.05)	(24.10)	(59.85)
100	2.05	3.85	4.90	6.10	9.00	2.15	5.55	27.15	27.80	51.25
	(6.00)	(4.95)	(5.65)	(6.55)	(9.35)	(6.00)	(7.30)	(29.40)	(29.35)	(52.00)
150	2.25	3.40	4.20	15.10	12.00	2.20	8.55	28.10	64.15	72.35
	(4.60)	(4.80)	(4.85)	(16.15)	(12.55)	(4.85)	(11.00)	(30.05)	(65.10)	(72.90)
200	2.60	4.35	5.65	14.55	85.50	3.45	18.85	48.85	64.15	99.10
	(6.00)	(6.15)	(6.15)	(15.25)	(85.85)	(7.15)	(22.70)	(50.80)	(65.20)	(99.10)
$\tilde{\Delta}_{HAC(QS)}$										
20	1.75	2.40	3.40	3.95	4.10	1.45	2.20	4.20	5.20	4.20
	(6.35)	(4.15)	(3.90)	(4.45)	(4.30)	(5.25)	(3.90)	(4.65)	(5.85)	(4.75)
50	4.35	3.30	2.50	4.55	5.80	3.95	2.45	2.85	11.65	30.35
	(10.25)	(4.30)	(3.00)	(5.25)	(6.15)	(9.90)	(3.05)	(3.55)	(12.10)	(31.20)
100	8.85	3.00	3.05	3.55	5.60	7.40	1.85	11.50	15.55	30.20
	(19.70)	(4.55)	(3.80)	(4.00)	(6.10)	(17.25)	(3.10)	(12.10)	(16.65)	(31.30)
150	11.90	4.05	2.85	4.95	7.05	10.50	2.10	6.80	32.00	53.95
	(25.55)	(5.15)	(3.25)	(5.20)	(7.55)	(22.70)	(2.95)	(7.55)	(33.20)	(55.00)
200	19.50	4.25	3.65	5.85	24.15	16.20	1.90	14.95	34.30	89.10
	(34.30)	(5.35)	(4.05)	(6.65)	(25.35)	(29.30)	(2.80)	(16.80)	(35.15)	(89.75)
$\tilde{\Delta}_{CSA}$										
20	2.10	2.85	4.65	4.80	4.80	1.75	2.85	5.75	5.65	5.25
	(7.80)	(6.35)	(6.00)	(5.60)	(5.05)	(7.50)	(5.85)	(6.65)	(6.45)	(5.70)
50	1.15	2.15	3.85	5.50	11.40	1.15	2.30	3.95	13.70	46.30
	(5.65)	(4.80)	(4.70)	(6.05)	(11.70)	(5.20)	(3.95)	(4.80)	(14.70)	(47.05)
100	1.10	2.75	4.30	4.30	7.15	1.15	2.65	12.25	17.00	36.25
	(5.25)	(5.05)	(5.30)	(5.20)	(7.60)	(5.05)	(5.55)	(14.40)	(18.35)	(37.65)
150	1.10	2.45	3.55	10.35	9.15	1.15	3.15	12.55	43.50	55.80
	(5.50)	(4.80)	(4.80)	(11.30)	(9.80)	(5.35)	(5.35)	(14.40)	(45.60)	(57.05)
200	1.25	3.00	4.20	9.75	77.60	1.40	3.60	23.05	44.80	98.30
	(6.20)	(4.55)	(5.20)	(10.65)	(78.50)	(6.10)	(6.85)	(25.45)	(46.50)	(98.35)
$\tilde{\Delta}_{oracle}$										
20	2.05	4.35	4.70	4.70	3.90	2.00	4.10	6.10	8.25	4.60
	(5.70)	(5.35)	(5.25)	(5.10)	(4.35)	(5.05)	(5.25)	(7.25)	(8.75)	(5.00)
50	1.85	3.55	4.10	5.15	4.30	2.05	5.15	6.05	16.15	49.15
	(4.60)	(4.80)	(4.60)	(5.60)	(4.85)	(4.25)	(5.95)	(6.65)	(16.65)	(49.65)
100	2.00	3.65	4.40	4.55	5.60	2.40	4.85	25.05	22.70	46.90
	(6.30)	(4.45)	(4.80)	(5.25)	(5.95)	(5.75)	(5.90)	(26.65)	(23.25)	(47.55)
150	2.30	3.45	3.65	4.75	4.00	2.35	10.00	25.90	57.70	56.20
	(4.85)	(4.55)	(4.10)	(5.35)	(4.05)	(4.80)	(11.40)	(27.70)	(58.60)	(56.95)
200	2.75	3.80	4.35	5.00	4.20	3.60	18.80	44.10	55.10	97.95
	(5.15)	(5.05)	(5.00)	(5.70)	(4.60)	(7.25)	(21.10)	(45.95)	(56.10)	(98.00)

Table 11: Specification 9 - Size and Power of Delta test with no cross-sectional dependence and normaly iid errors with heteroskedasticity.
Bandwidth for $\tilde{\Delta}_{HAC}$ is automatically selected following Newey and West (1994). Only contemporaneous cross-sectional averages are added for $\tilde{\Delta}_{CSD}$. Results for the small sample adjusted $\tilde{\Delta}_{adj}$ are given in parenthesis. 4 exogenous regressors. β_1 is heterogenous under the null and the alternative. For a definition of the DGP see Section 5. In $\tilde{\Delta}_{oracle}$ the correct coefficients are partialled out. For a definition of the DGP see section 5.